Interactive Computer Theorem Proving

Lecture 1: Why ICTP?

CS294-9
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About Me

- 4th year CS PhD student in programming languages
- Started doing interactive computer theorem proving in Spring 2004, as part of the Open Verifier project
- Now it's the main focus of my research.
- Specifically, developing programming language tools with proofs of correctness

This Class

- A practical perspective on computer theorem proving
- Designed to be accessible to anyone who's taken a basic logic and discrete math class
- Experience with functional programming is a plus
  - Scheme/Lisp good, ML/Haskell better :-)

Administrivia

- Usually meet only on Thursdays
- One homework assignment a week during the first half of the course
  - Exercises using Coq (a proof assistant)
- For people taking the class for 3 units, a standard research project in a small group
  - Probably some application of interactive computer theorem proving

Administrivia II

- No required text, but the *Coq'Art* book is a useful reference
  - We have a few copies that we can loan out as needed
- This class probably won't satisfy any CS PhD breadth requirement, but see us if this is a problem for you.

What is a Proof?

- Proof by example
  - The author gives only the case n = 2 and suggests that it contains most of the ideas of the general proof.
- Proof by intimidation
  - "Trivial."
- Proof by vigorous handwaving
  - Works well in a classroom or seminar setting.
- Proof by cumbersome notation
  - Best done with access to at least four alphabets and special symbols.
- Proof by exhaustion
  - An issue or two of a journal devoted to your proof is useful.

[excerpt from a popular e-mail forwarding bonanza]

Classical Motivations

- Mathematicians and philosophers want to formalize their reasoning processes.
- Interest in formal methods driven by how difficult it is to be sure that a mathematical system corresponds to our intuitions.
- Want to come up with tiny but very expressive systems to study very carefully.

Don't Worry!

- This class is not about sitting around debating the metaphysics of “1 + 1 = 2.”
- We'll focus on a variety of practical applications of theorem proving technology.
- ...not that those philosophers didn't have some ideas that have turned out to be very practical. ;-)

Correctness is Nice

- Expensive mistakes
  - Pentium FDIV bug
  - Ariane rocket crash
  - etc.
- Programming language semantics
  - The POPLmark Challenge

The Age of “Security”

- The Internet isn't a friendly place anymore.
- “We want to make sure our software can't be exploited.”
  - Verification of cryptographic protocols, etc.
- “We want to use software written by someone we don't trust.”
  - Proof-carrying code

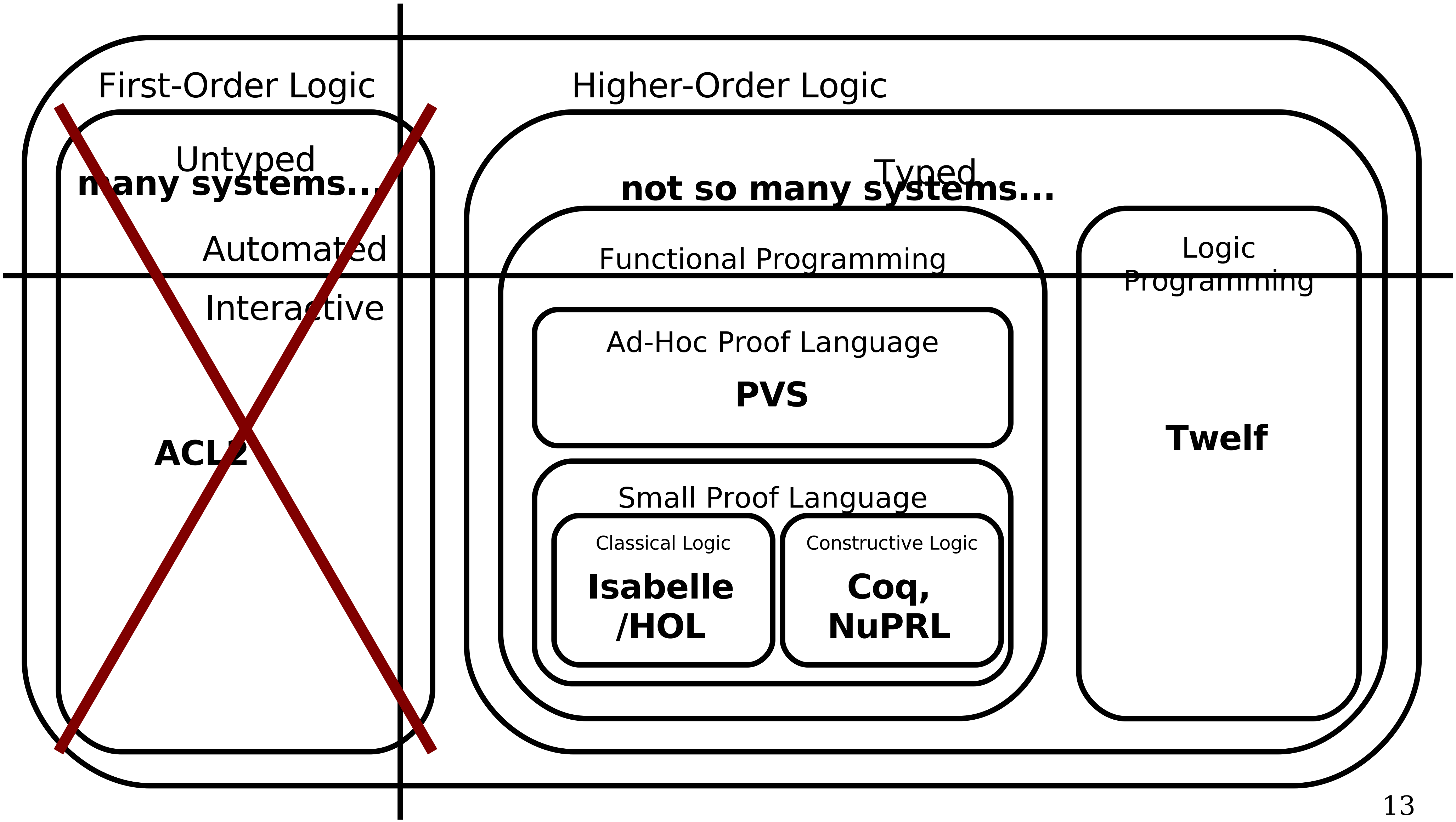
Software Engineering

- Developing programs and their correctness proofs simultaneously is an alternative to test-based development.
- The more intricate the system, the more likely it is that proof is more effective than testing.
- Exactly how to do this is a very active research topic today.

Goals for This Course

- Learn how to use the **Coq proof assistant** to:
  - Formalize most any kind of math
  - Formalize theory related to your research
  - Develop practical functional programs with total correctness proofs
- Learn exactly what it means for a proof to be rock solid, so that even a computer believes it.

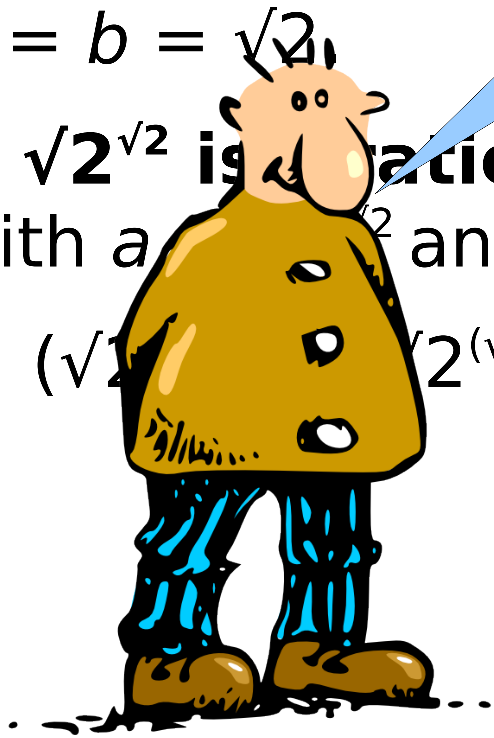
The World of Computer Theorem Proving



Constructi… …ic?

OK, but how does that help me **compute** *a* and *b*?

- *Theorem*: There exis… …b such that a^b is rational
- **If $\sqrt{2}^{\sqrt{2}}$ is rational**, …en we have the theorem with $a = b = \sqrt{2}$.
- **If $\sqrt{2}^{\sqrt{2}}$ is …rational**, then we have the theorem with a … and $b = \sqrt{2}$.
  - ($\sqrt{2}$… …$\sqrt{2}^{(\sqrt{2}\sqrt{2})} = \sqrt{2}^2 = 2$



A Constructive Proof

- *Theorem*: Every degree-one rational polynomial $y = mx + b$ has a rational root if m is not 0.
- *Proof*: $-b/m$ is the answer, because:
  - $m(-b/m) + b = -b + b = 0$

```
rational root(rational m, rational b) {
   return -b / m;
}
```

- *Precondition*: m is not 0.
- *Postcondition*: The return value is a root of $y = mx + b$.

An Even Nicer Idea

- *Theorem*: Every Java program has an equivalent x86 machine language program.
- By choosing a suitable *constructive logic*, we guarantee that **any proof of this theorem can be converted into a genuine Java compiler**!
- By using a generic *program extraction mechanism*, we get the “free” theorem that our compiler preserves the semantics of programs.
  - …which saves us a huge amount of testing.

Example: Alias Analysis

```
int x, y;
int *p;

p = &y;
x = 1;
*p = 2;

return x;
```

Compiler Optimizer

```
return 1;
```

Empty intersection!

The path *x* only ever denotes elements of {&*x*}.
The path *y* only ever denotes elements of {&*y*}.
The path **p* only ever denotes elements of {&*y*}.

Andersen's Analysis

L: `x = new` $\qquad$ `x = y`

$L \in PT(x)$ $\qquad$ $PT(y) \subseteq PT(x)$

`x = *y` $\qquad$ `*x = y`

$\forall\, v \in PT(y), PT(v) \subseteq PT(x)$ $\qquad$ $\forall\, v \in PT(x), PT(y) \subseteq PT(v)$

- Ignore order of instructions in the program.
- Treat all allocations occurring in the same instruction as if they allocated the same object.
- For each program variable *x*, build a set *PT*(*x*) that overapproximates the locations *x* might point to.
- Generate and solve a set of constraints over the *PT* sets.

Andersen in Coq

- A Coq implementation of Andersen's Analysis for this toy language, with a *proof of total correctness*
- Not quite so convoluted as you may be expecting from the slides on constructive logic, thanks to connections between proofs and functional programs that I haven't presented yet

But First...

How would you prove the correctness of Andersen's Analysis?

(if you had to convince someone who can only be convinced by a series of “obvious” steps)

Conclusion

- The full code of this example is available on the course web site.
- HW0 is posted
  - Install Coq and make sure you can run some simple examples through it.
- Next lecture: Revisiting freshman logic class
  - Natural deduction and interactive Coq proofs of theorems in propositional and first-order logic