Thoughts on Programming with Proof Assistants

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PLPV Workshop
Not Ready for Prime Time?

Proof assistants like Coq are useful for doing math, but they're far too inconvenient for serious dependently typed programming!

Cons
- No imperativity, general recursion, or exceptions
- Very primitive dependent pattern matching

Pros
- Easy to combine programming with tactic-based proving
- A mature set of tools for proof organization and automation

This Talk: Capsule summary of my experiences implementing Proof-Carrying Code-style program verifiers in Coq using dependent types to guarantee total correctness.
Mixing Programming with Tactics

Definition isEven : forall n, [even(n)].
  refine (fix isEven (n : nat) :
    [even(n)] :=
    match n return [even(n)] with
    O -> Yes
    | S O -> No
    | S (S n) -> proof <­ isEven n; Yes);
  auto.
Qed.

Step 1. Declare the function we mean to write as a proof search goal.
Step 2. Give the “program part” of the implementation.
Step 3. Generate the “proof part” of the implementation using tactics.

Result: A term in the Calculus of Inductive Constructions
Imperativity?

Pure functional data structures worked well for all of the situations I encountered.

Non-termination and general recursion?
The kinds of program analysis algorithms I needed were naturally primitive recursive.

Exceptions?
Failure monads provide a cleaner alternative to “exceptional” uses of exceptions.

Fancy dependent pattern matching?
Sticking to refinement types, vanilla pattern matching is good enough.
Reflective Proofs

Theorem check : \( \forall (p : \text{program}), \) 
\[ \text{verify } p = \text{true} \] 
\( \to \) safe \( p \).

(check \( p \) (refl_equal (verify \( p \)))

Typecheck

\( \text{verify } p = \text{verify } p \)

Computational reduction \( \text{via definitional equality} \)

\( \text{verify } p = \text{true} \)
Other Benefits

- Module system
- Lots of pre-written proof-generating decision procedures
- Expressive tactical language
- Extensible goal-directed proof search mechanism
- Extraction to OCaml (and from there to fast native code)
Conclusion

- Consider using Coq for your next dependently typed program if large non-syntax-directed proofs are a large part of it.
- It seems worthwhile to keep in mind potential overlaps between programming environments and proof assistants in developing new PLPV tools.

For more info: See my talk at ICFP next month!